



Brockswood Primary and Nursery School Calculation Policy

Date: January 2019
Review: January 2020

Progression in Multiplication and Division

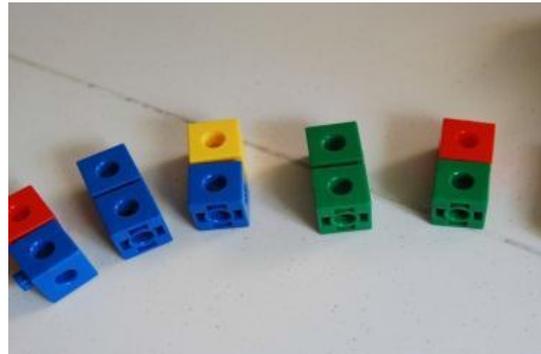
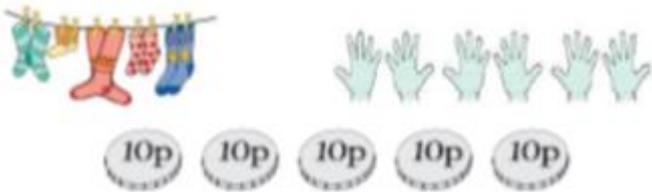
Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

| | | | |
|-------|------|------|------|
| Part | Part | Part | Part |
| Whole | | | |

Multiplication Phase 1: Strategies

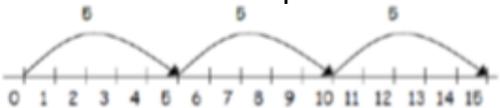
Counting in Multiples

- Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups. Children will look at doubling.



Repeated addition

3 times 5 is $5 + 5 + 5 = 15$ or 5 lots of 3 or 5×3
Children learn that repeated addition can be shown on a number line.

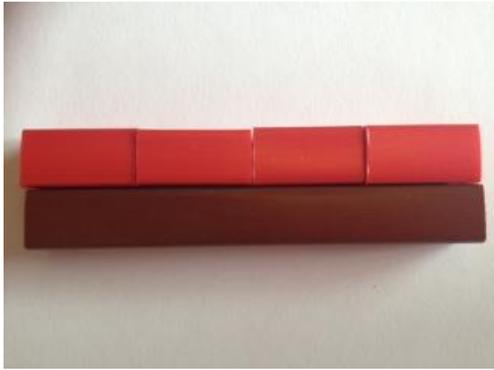


Children learn that repeated addition can be shown on a bead string.



Children also learn to partition totals into equal trains using Cuisenaire Rods





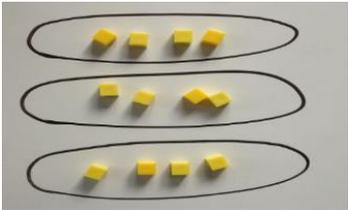
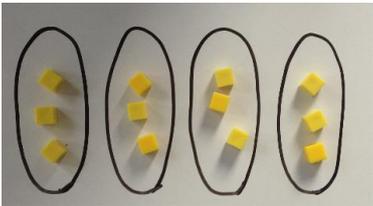
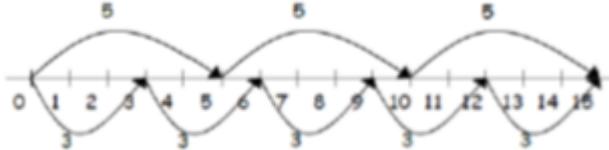
Focus on verbalising thinking:
 e.g.
 'eight is two taken four times'
 'two taken four times is eight'
 'eight equals four times two'
 'there are four twos in eight'

Multiplication Phase 2: Strategies

Commutativity

Children learn that 3×5 has the same total as 5×3 . This can be shown on the number line.

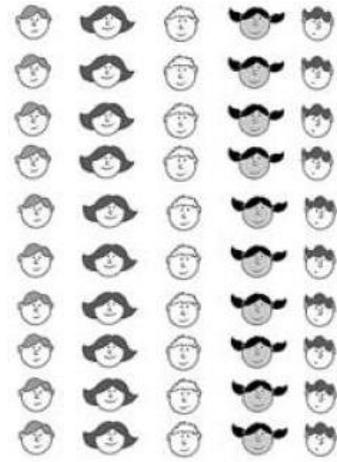
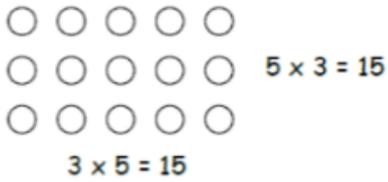
$3 \times 5 = 15$
 $5 \times 3 = 15$



Arrays

Children learn to model a multiplication calculation using an array. Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.

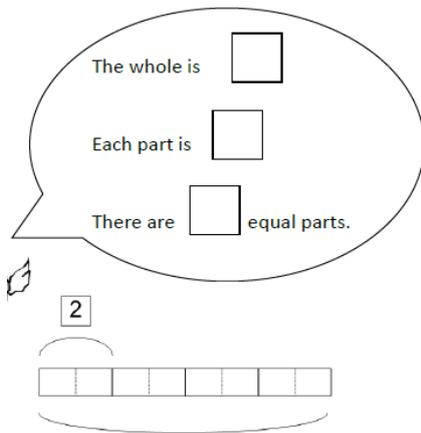




Revisiting Part-Part Whole model

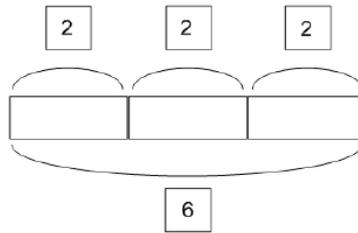
This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the size of the parts and the number of parts.

Use your Cuisenaire rods to replicate the bar models



What multiplication and division equations can you write for each bar model?

Prove that the equations are correct using a bead string.



| | | | | |
|----------------------|----------|----------------------|-----|----------------------|
| <input type="text"/> | \times | <input type="text"/> | $=$ | <input type="text"/> |
| <input type="text"/> | \div | <input type="text"/> | $=$ | <input type="text"/> |

Multiplication

Phase 3: Strategies

Partitioning for multiplication

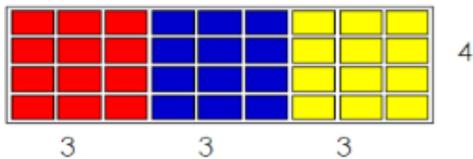
Arrays are also useful to help children visualise how to partition larger numbers into more useful representation.

$$9 \times 4 = 36$$



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

$$9 \times 4 =$$

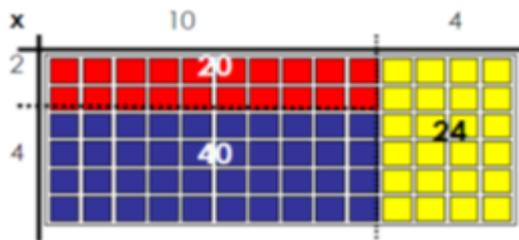


Which could also be seen as

$$9 \times 4 = (3 \times 4) + (3 \times 4) + (3 \times 4) = 12 + 12 + 12 = 36$$

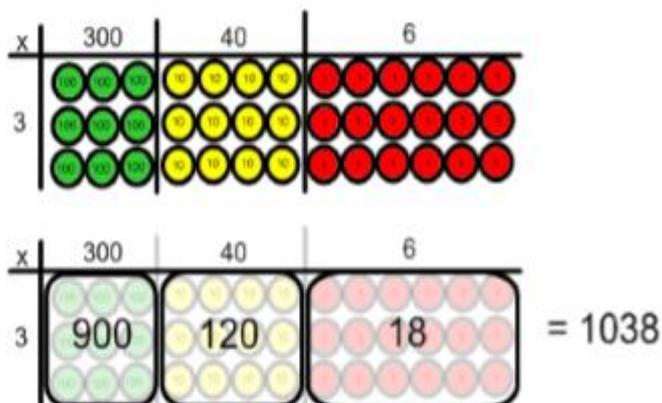
$$\text{Or } 3 \times (3 \times 4) = 36$$

$$\text{And so } 6 \times 14 = (2 \times 10) + (4 \times 10) + (4 \times 6) = 20 + 40 + 24 = 84$$



Grid Method

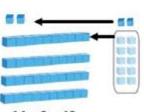
This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total.



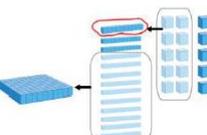
$$23 \times 4 = 92$$

| | | | |
|---|----|----|--|
| x | 20 | 3 | |
| 4 | 80 | 12 | $\begin{array}{r} 80 \\ + 12 \\ \hline 92 \end{array}$ |

| | | | |
|---|---|---|--|
| x | 10 | 4 | |
| 3 |  |  | |

| | | | |
|---|----|----|---|
| x | 10 | 4 | |
| 3 | 30 | 12 |  |

$14 \times 3 = 42$

| | | | |
|---|---|---|---|
| x | 40 | 5 | |
| 3 |  |  |  |

Short Written Method

This method leads on from the grid method by linking multiplying the ones and then the tens. They will then begin to make links to a more efficient method where the tens are carried.

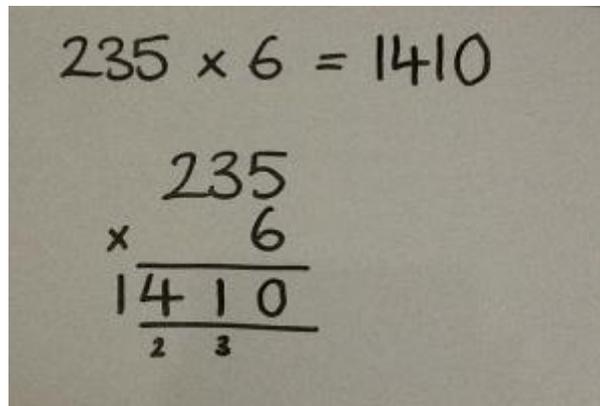
$$23 \times 4 = 92$$

| | |
|------|----------|
| 23 | |
| x 4 | |
| 12 | (4 x 3) |
| + 80 | (4 x 20) |
| 92 | |

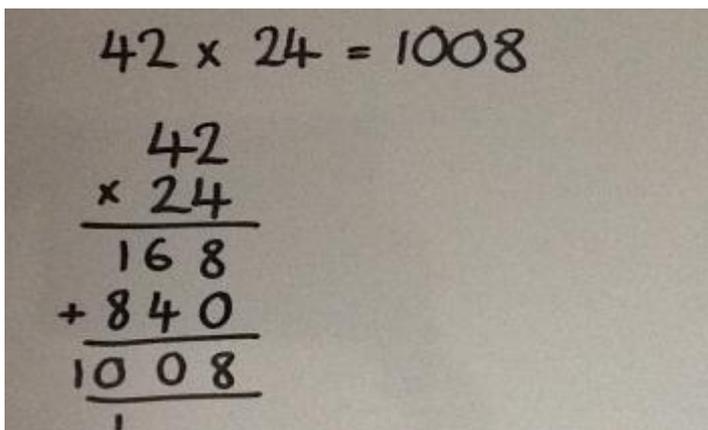
Multiplication Phase 4: Strategies

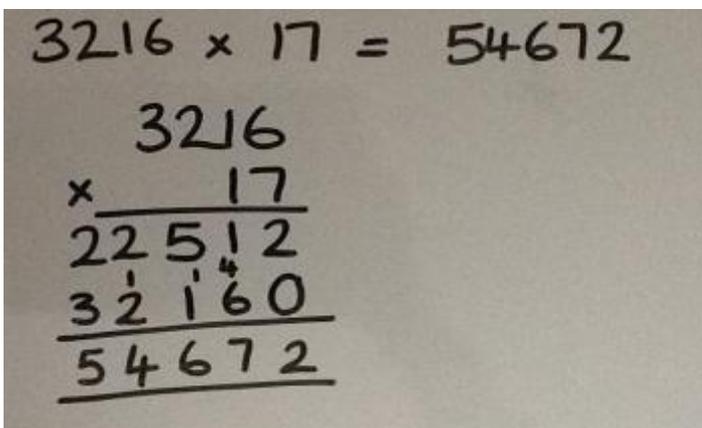
Using efficient methods

Children will be taught to multiply numbers (HTUxU) using the formal written method.


$$235 \times 6 = 1410$$
$$\begin{array}{r} 235 \\ \times \quad 6 \\ \hline 1410 \\ \hline \end{array}$$

Once children are secure, this moves onto (TU x TU, HTU x TU and ThHTU x TU)


$$42 \times 24 = 1008$$
$$\begin{array}{r} 42 \\ \times 24 \\ \hline 168 \\ + 840 \\ \hline 1008 \\ \hline \end{array}$$

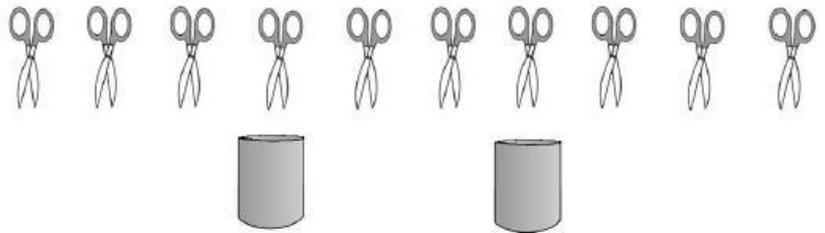
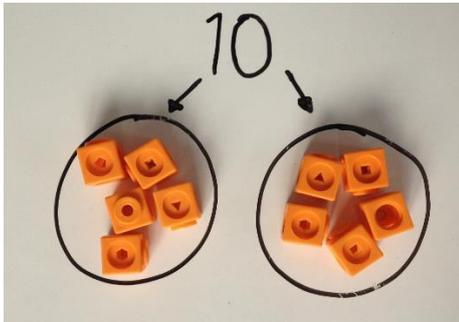

$$3216 \times 17 = 54672$$
$$\begin{array}{r} 3216 \\ \times \quad 17 \\ \hline 22512 \\ 32160 \\ \hline 54672 \\ \hline \end{array}$$

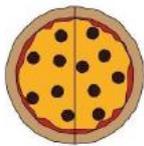
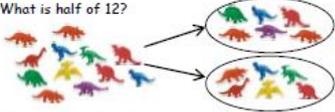
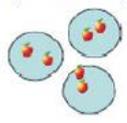
Division

Phase 1: Strategies

Sharing

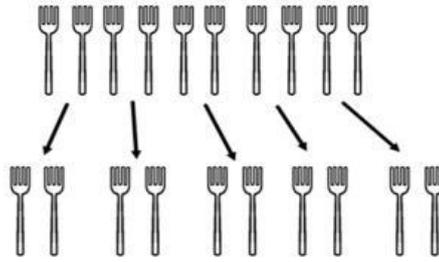
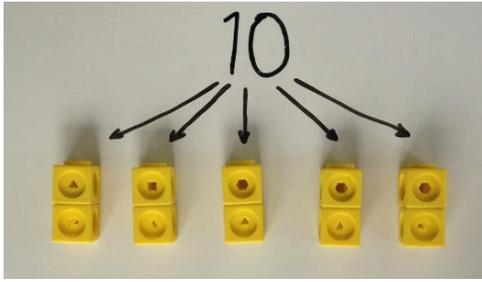
If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot. Children will solve problems of halving and sharing.



| |
|---|
| Share 12 cakes between 3 people equally:  |
| Can you cut the pizza in half?  |
| What is half of 12?  |
| Can you share 6 apples between 3 plates?  |
| $8 \div 2 = 4$  |

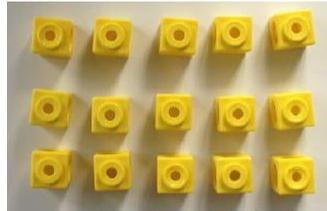
Grouping

If we have ten forks and we put them into groups of two, there are 5 groups. Children will understand the concept of equal groups.



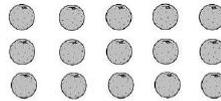
Part-Part Whole

Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication. Pupils use arrays of concrete manipulatives and images of familiar objects to find division equations. They begin to use dot arrays to develop a more abstract concept of division.



$$15 \div 5 = \boxed{3}$$

$$15 \div 3 = \boxed{5}$$



Write the division equations that the array represents.

$$20 \div 4 = \boxed{}$$



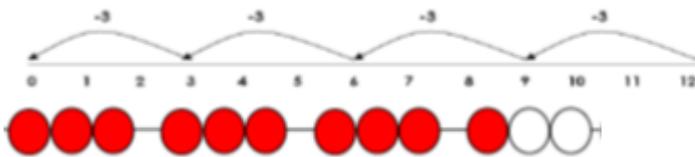
$$20 \div 5 = \boxed{}$$

Division Phase 2: Strategies

Repeated Subtraction

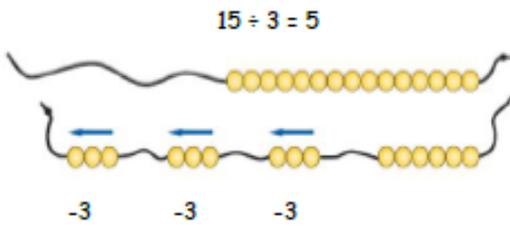
Children will use number beads and number lines.

$$12 \div 3 = 4$$



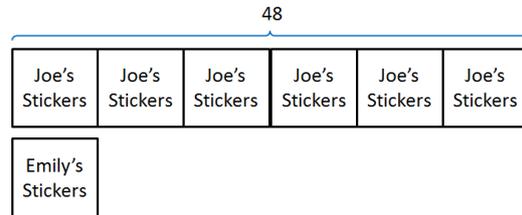
The bead string helps children with interpreting division calculations, recognising that $12 \div 3$ can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations. This shows where the bar model can be introduced.



Joe has 6 times as many stickers as Emily. Joe has 48 stickers. How many stickers does Emily have?

We know that Joe has 48 stickers in total.



Grouping involving remainders

Children move onto calculations involving remainders.

$13 \div 4 = 3 \text{ r}1$



Or using a bead string see above.

Inverse operations

Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

$3 \times 4 = 12$

$4 \times 3 = 12$

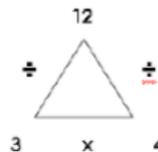
$12 \div 3 = 4$

$12 \div 4 = 3$

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

$\square \times 5 = 20$ $3 \times \Delta = 18$ $\bigcirc \times \square = 32$

$24 \div 2 = \square$ $15 \div \bigcirc = 3$ $\Delta \div 10 = 8$



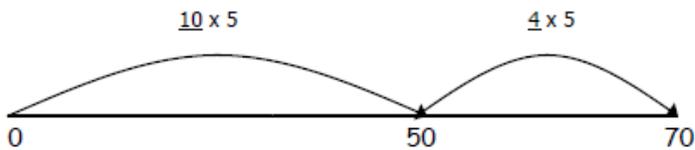
Division

Phase 3: Strategies

Chunking on a numberline

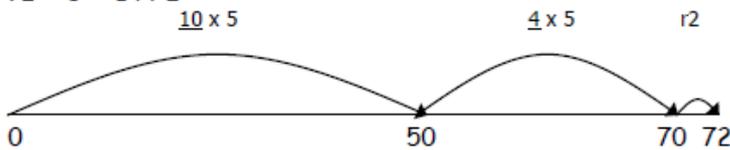
$$70 \div 5 = 14$$

Moving onto chunking on a numberline. Relate to the inverse of multiplication. Ask, how many 5s in 70? 10 lots of 5, 4 lots of 5. How many lots of 5 altogether? 14.



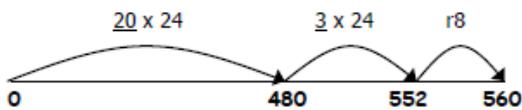
Division with remainders:

$$72 \div 5 = 14 \text{ r } 2$$



HTU \div TU

$$560 \div 24 = 23 \text{ r } 8$$



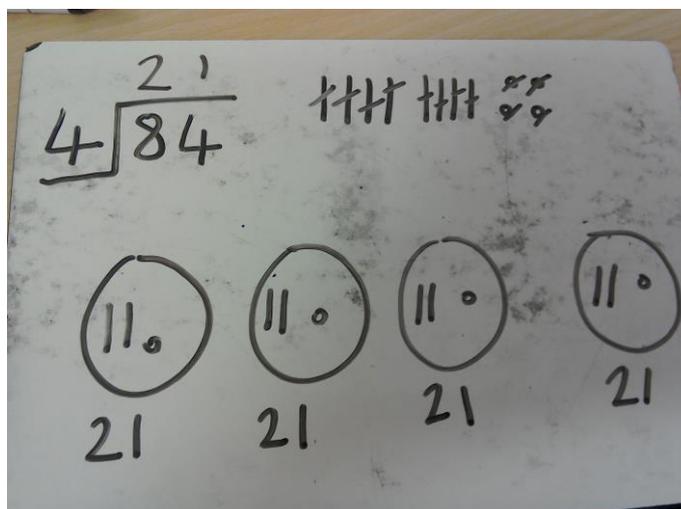
Tip: Children can jot down coin multiplication facts to give them a bank of facts to use on their number line.

| |
|------------------------|
| $1 \times 24 = 24$ |
| $2 \times 24 = 48$ |
| $5 \times 24 = 120$ |
| $10 \times 24 = 240$ |
| $20 \times 24 = 480$ |
| $50 \times 24 = 1200$ |
| $100 \times 24 = 2400$ |

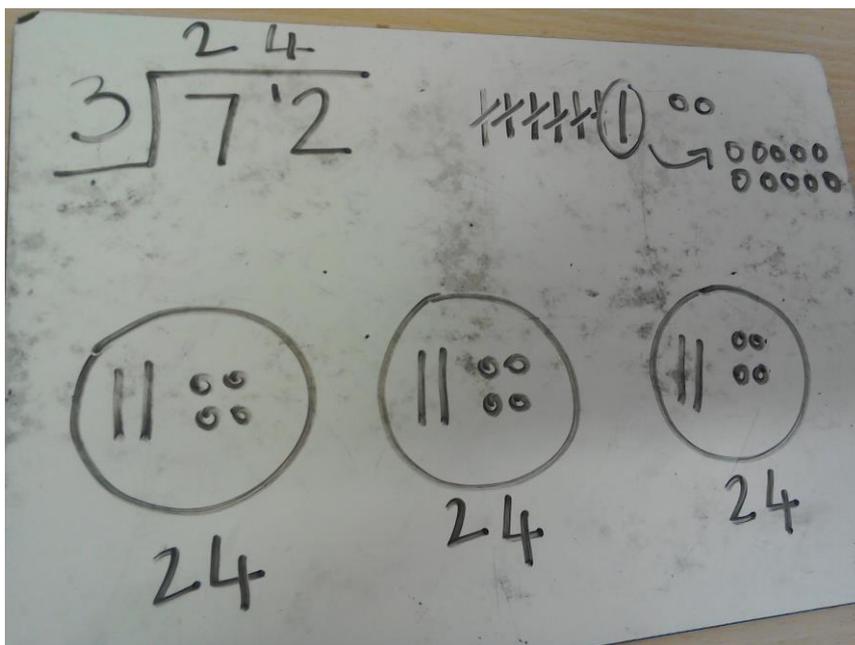
Coin Multiplication takes a given number and multiplies it by 1, 2, 5, 10, 20, 50 and 100.

Using Base Ten Equipment

Children start with dividing a two digit number by 2,3 and 4, where no regrouping is required. This is shown below using the base ten equipment below.



Conceptual understanding is developed by dividing a two digit number by a one digit number, where regrouping is required or where there is a remainder.



Short Division (Bus Stop method)

$$\begin{array}{r} 27 \\ 3 \overline{) 821} \end{array}$$

$$\begin{array}{r} 023 \\ 24 \overline{) 5572} \end{array}$$

$$\begin{array}{r} 023r8 \\ 24 \overline{) 55680} \end{array}$$

Children can then progress to finding an answer with a decimal:

$$\begin{array}{r} 22.5 \\ 6 \overline{) 1315.30} \end{array}$$

Gradation of difficulty (short division)

1. $TO \div O$ no exchange no remainder
2. $TO \div O$ no exchange with remainder
3. $TO \div O$ with exchange no remainder
4. $TO \div O$ with exchange, with remainder
5. Zero in the quotient e.g. $816 \div 4 = 204$
6. As 1-5 $HTO \div O$
7. As 1-5 greater number of digits $\div O$
8. As 1-5 with a decimal dividend e.g. $7.5 \div 5$ or $0.12 \div 3$
9. Where the divisor is a two digit number

Gradation of difficulty with remainders (short division)

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.

e.g.:

- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

1. Whole number remainder
2. Remainder expressed as a fraction of the divisor
3. Remainder expressed as a simplified fraction
4. Remainder expressed as a decimal